Discrete Random Variables
Day 1

What is a Random Variable?

Every probability problem is equivalent to drawing something from a bag (perhaps more than once)

Like…

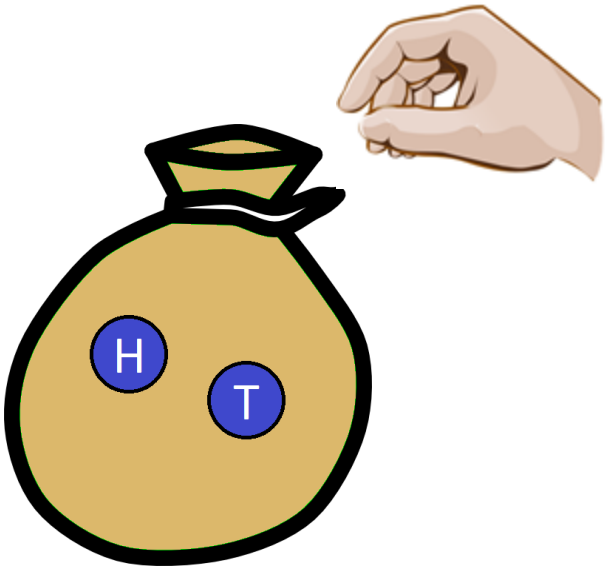
Flipping a coin 3 times

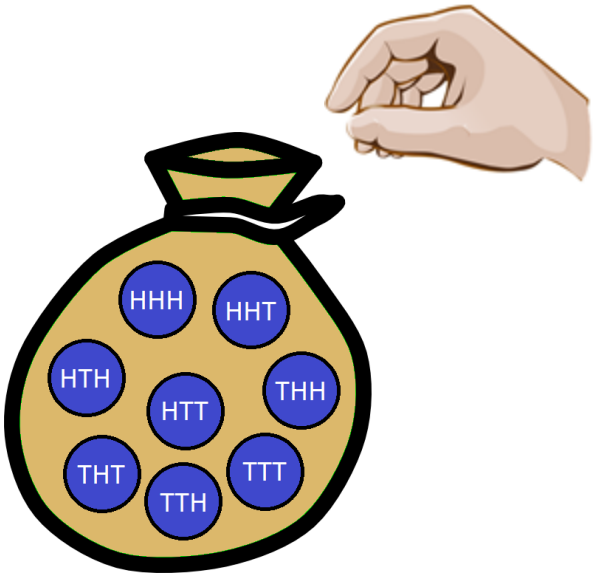
is equivalent to

Drawing a ball 3 times from the bag below (with replacement)

is equivalent to

Drawing 1 ball from the bag below





What is a Random Variable?

We want every probability problem to be equivalent to drawing a NUMBER from a bag (and only once).

Definition: A random variable is a way of relabeling all of the outcomes of an experiment with NUMBERS.

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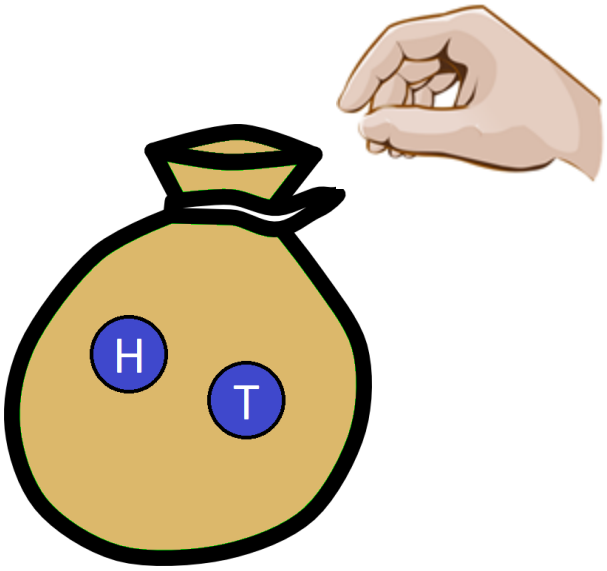

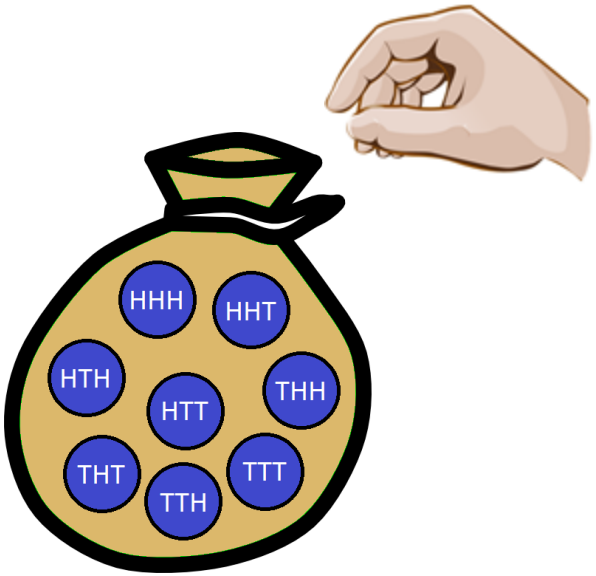

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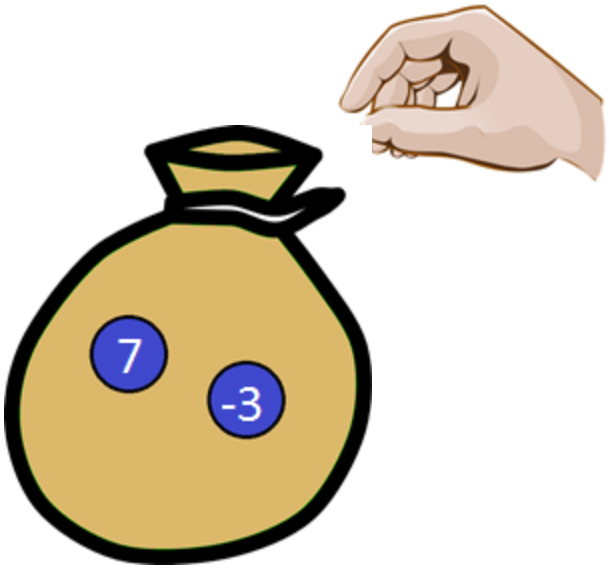


Defining Random Variables

Ex 1: Define some random variables on the following experiments:

a) Experiment = Flip a single coin 4 times

b) Experiment = Draw a single card from a standard poker deck

c) Experiment = Draw 2 cards from a standard poker deck one by one with replacement

d) Experiment = Draw 2 cards from a standard poker deck one by one without replacement

Probability Distributions of Random Variables

- Every random variable has a probability distribution
- The probability distribution tells you the probability for each value of the random variable
- For a discrete random variable, the probability distribution is a table (or a histogram)
- To calculate a probability distribution for a random variable, GO BACK TO THE SAMPLE SPACE

2 Requirements for a Probability Distribution for a Discrete Random Variable

1) For every value x of the random variable X,

$$0 \leq P(X = x) \leq 1$$

2) $$\sum P(X = x) = 1$$

Ex 2:

Experiment = Roll a pair of dice

Random Variable X = Total of the numbers on the dice

a) Find the probability distribution of X
(as a table and as a histogram)

b) Verify that your answer to part (a) satisfies the 2 requirements of a probability distribution

Ex 3:

Experiment = Draw a single card from a standard poker deck

Suppose you make a bet with your friend where you…

Win $10 if you draw the ace of spades

Win $5 if you draw any other ace

Win $2 of you draw any other spade

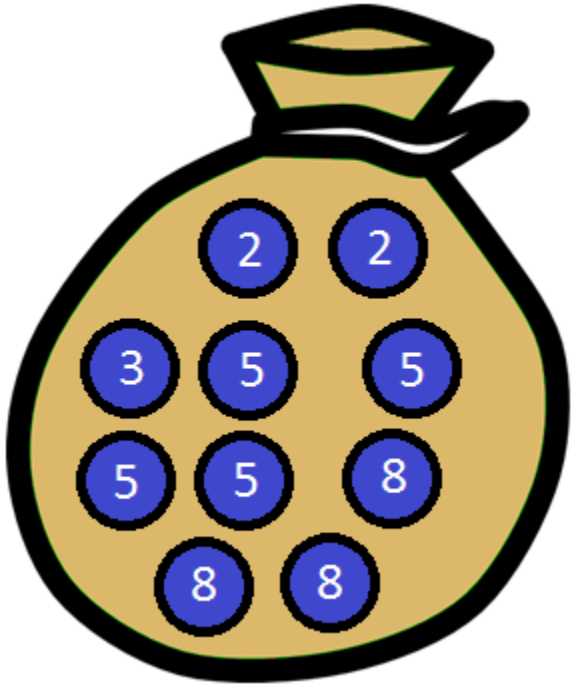
Lose $1 of you draw anything else

Random Variable X = Amount of money you win when playing this game once

a) Find the probability distribution of X
(as a table and as a histogram)

b) Verify that your answer to part (a) satisfies the 2 requirements of a probability distribution

Expected Value, Standard Deviation, and Variance of a Discrete Random Variable



x	$P(X = x)$
2	$\frac{2}{10}$
3	$\frac{1}{10}$
5	$\frac{4}{10}$
8	$\frac{3}{10}$

Expected Value, Standard Deviation, and Variance of a Discrete Random Variable

Expected Value

$$\mu = E(X) = EV(X) = \sum xP(X = x)$$

Standard Deviation

$$\sigma = SD(X) = \sqrt{\left[\sum x^2 P(X = x)\right] - \mu^2}$$

Variance

$$\sigma^2 = VAR(X) = \left[\sum x^2 P(X = x)\right] - \mu^2$$

Expected Value, Standard Deviation, and Variance of a Discrete Random Variable

Note:

1) The reason why we are using the symbols μ, σ, σ^2 Is because we are pretending that the bag of numbers is POPULATION data, not sample data.

2) The formula $\sigma = SD(X) = \sqrt{\left[\sum x^2 P(X = x)\right] - \mu^2}$ comes from our old formula for standard deviation of the POPULATION

$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}}$ not $s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{n\sum x^2 - (\sum x)^2}{n(n-1)}}$

SD of population data SD of sample data

Ex 4: You and a friend are betting on the roll of a die. Specifically you will lose $1 if you roll a 1, 2, or 3, you will lose $2 if you roll a 4 or 5, and you will win $8 if you roll a 6. Let the random variable X denote the amount of money you win when playing this game once.

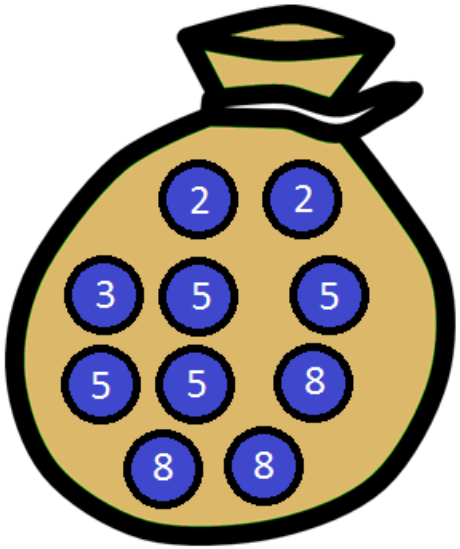
a) Find the probability distribution of X

b) Find the expected value, standard deviation and variance of X

c) Explain the meaning of the expected value you obtained in part (b)

d) Is this a good bet for you? Or for your friend?

Ex 5: You are going to draw a single ball from the bag below once. Let X denote the number on the ball that you drew.



a) Find the probability distribution of X

b) Find the expected value, standard deviation and variance of X

c) Explain the meaning of the expected value you obtained in part (b)

Hint: Some of this calculation was already done earlier today

Ex 6: In this example we are going to analyze 2 different bets in roulette.

Bet 1: You bet \$100 on red. Let X denote the amount of money you win when you make this bet once.

Bet 2: You bet \$100 on the number 28. Let Y denote the amount of money you win when you make this bet once.

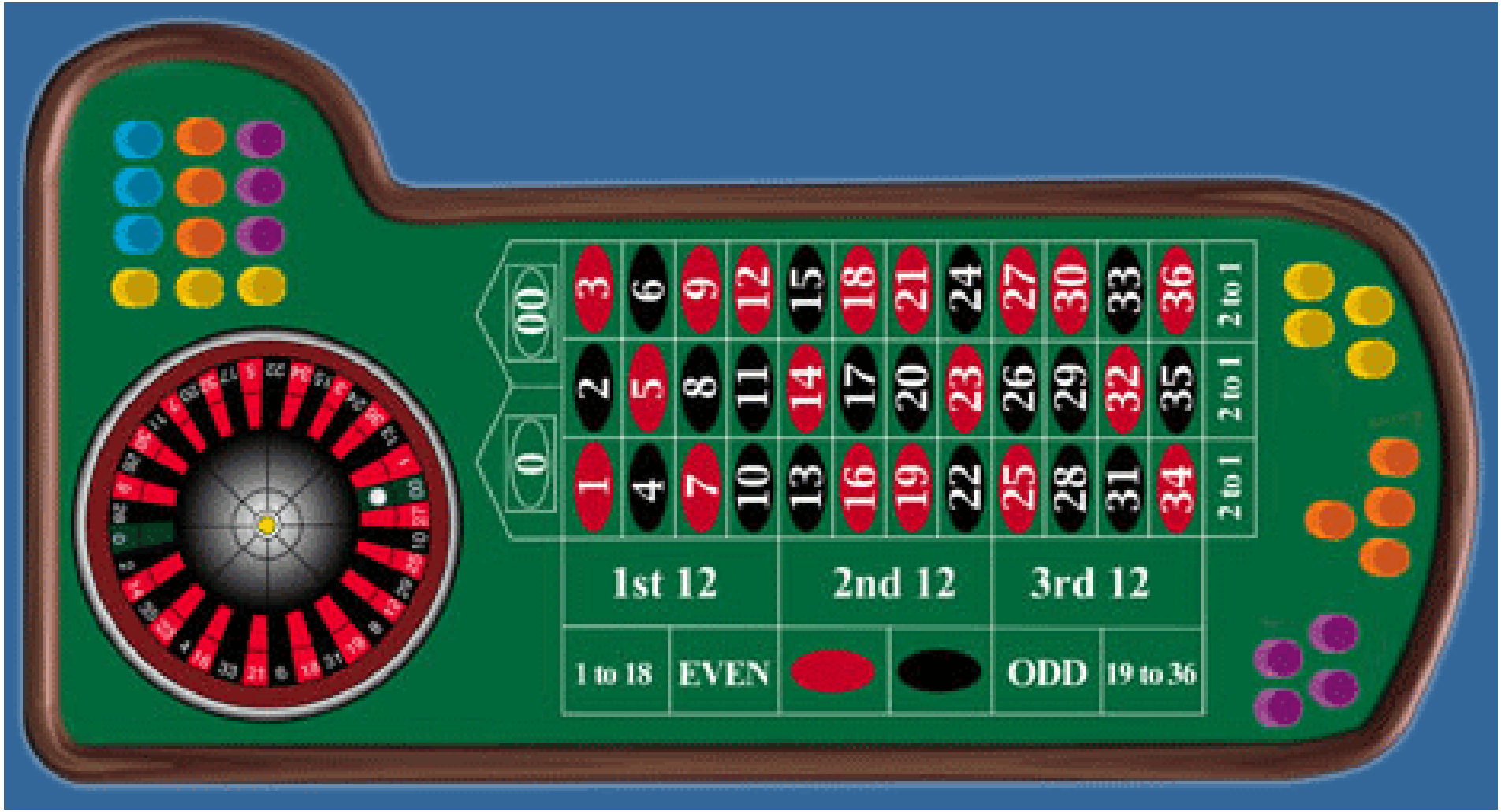
a) Find the probability distributions of X and Y

b) Find the expected values, standard deviations and variances of X and Y

c) Explain the meaning of the expected values you obtained in part (b)

d) Discuss which is a better bet.

Ex 6 (picture):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3 6 9 12 15 18 21 24 27 30 33 36
2 5 8 11 14 17 20 23 26 29 32 35
1 4 7 10 13 16 19 22 25 28 31 34
2 to 1
2 to 1
2 to 1
1st 12
2nd 12
3rd 12
1 to 18
EVEN
ODD
19 to 36